UNCLASSIFIED

AD NUMBER
AD817515
LIMITATION CHANGES
TO: **Approved for public release; distribution is unlimited.**
FROM: **Distribution authorized to U.S. Gov't. agencies and their contractors; Critical Technology; MAY 1967. Other requests shall be referred to Air Force Materials Laboratory, ATTN: MAM, Wright-Patterson AFB, OH 45433.**
AUTHORITY
AFML ltr dtd 12 Jan 1972

THIS PAGE IS UNCLASSIFIED

AD0817515

FDDS-Merkle

FDDA

AFML-TR-67-149

FILE

67-149

RELIABILITY ANALYSIS BASED ON TIME TO FIRST FAILURE

ALFRED M. FREUDENTHAL

COLUMBIA UNIVERSITY

TECHNICAL REPORT AFML-TR-67-149

MAY 1967

AIR FORCE MATERIALS LABORATORY
RESEARCH AND TECHNOLOGY DIVISION
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

50th Anniversary
1917 1967
AIR FORCE MATERIALS LABORATORY
Materials Research in the Air Force

NOTICES

400 - June 1967 - CO192 - 37- 915

AFML-TR-67-149

RELIABILITY ANALYSIS BASED ON TIME TO FIRST FAILURE

ALFRED M. FREUDENTHAL

FOREWORD

This report was prepared by Dr. A. M. Freudenthal, New York, N. Y. under USAF Contract AF 33(615)-3430. The contract was initiated under Project No. 7351 , "Metallic Materials", Task No. 735106, "Behavior of Metals". The contract was administered by the Ohio State University Research Foundation. The work was monitored by the Metals and Ceramics Division, AF Materials Laboratory, Research and Technology Division, Air Force Systems Command, Wright Patterson Air Force Base, Ohio, under the direction of Mr. W. J. Trapp.

This report covers the period of work, April 15 to December 31, 1966.

This technical report has been reviewed and is approved.

Manuscript released by the author February 1967 for publication.

W. J. Trapp

W. J. TRAPP
Chief, Strength and Dynamics Branch
Metals and Ceramics Division
Air Force Materials Laboratory

ABSTRACT

The new approach to structural reliability analysis based on order statistics and the expected time to first failure in a fleet of specified magnitude, that has been first proposed in a previous report AFML-TR-66-37 is developed in more detail and applied to structures subject to progressive fatigue damage. It is shown that the concept of fatigue sensitivity developed in WADD Tech. Rep. 61-53 is improved by relating it to the expected time to first failure rather than to the expected time to failure. The modified fatigue sensitivity factor becomes an effective parameter for the correlation of ultimate load and fatigue design and for the classification of fatigue sensitive structures.

TABLE OF CONTENTS

ILLUSTRATIONS

I. Introduction

A fundamental difference in the approach to the life and reliability estimate of easily maintainable, multi-element electronic systems and of major structures the performance of which depends on a relatively small number of critical, and essentially "non-maintainable" elements results from:

(a) the different implication of the concept of "failure",

(b) the large difference in the possible number of test replications to provide an estimate of "mean life", and

(c) the basic difference in the procedures of reliability demonstration.

In electronic systems failure in operation is considered a contingency, reducible by a program of periodic replacement of components to an acceptable condition defined by a small number of failures. The mean time to failure of elements, on the reliable estimate of which the effective design of the replacement program depends, is obtained from tests with a sufficiently large number of replications. For a major structure failure is not a contingency but an emergency. Since the consequences of even a single failure are usually severe, the concept of "mean life", with its tacit implication of a high percentage of failed structures has no relevance to design. It can also not be reliably estimated from the single test or the very small number of test replications that are technically and economically feasible. Therefore the approach to reliability demonstration is quite different: in electronic systems reliability demonstration of the whole system by computation based on experimentally demonstrated component reliability

is acceptable, while even the crudest estimate of the reliability of a full-scale structure requires a life test of this structure itself, supplemented by knowledge, accumulated in multiple tests of large structural elements, of the expected range of scatter.

This difference in approach must be reflected in a difference in methodology. While in reliability analysis of electronic systems linear functional models of utmost simplicity, such as the simple chain or parallel chains, form the substructure upon which probabilistic models of different levels of sophistication are superimposed, functional models representing structural failure are by the nature of the physical process highly nonlinear and redundant. Sophistication in the superimposed probability model would therefore compound the difficulties of analysis to an extent that does not seem warranted by the obtainable results in view of the severely limited number of experimental data available for their verification. Since under such conditions reliability demonstration must rely on a combination of full scale testing and performance during early life, preferably under operational conditions of more than normal severity, the expected time to the first failure[1] seems to be a concept that is more relevant to the life estimate and the reliability demonstration of major structures, in the design of which the avoidance of even a single catastrophic failure represents the overriding design criterion, than the conventional concepts of expected (mean, median or modal) life and scatter factor. Obviously, failure need not necessarily be defined as catastrophic, but can refer to a designated level of damage. In this case the expected time to the second failure as well as the expected interval between first and second failures become useful as additional characteristics from which not only the "expected life" might be estimated, but which might serve to verify the validity of various possible assumptions concerning the form of the distribution function of fatigue life.

The concepts of the expected time to the first failure as well as that of the interval between the first and second failures or that of the time to the second failure also have the practical advantages that

(i) they can be actually demonstrated in operation, which is impossible for the mean time to failure,

(ii) they are strongly affected by differences between different distributions of life so that they automatically reflect the differences between different failure mechanisms which lead to different distributions of times to failure, and

(iii) they depend on the size of the group of structures to be considered.

It seems, therefore, to be of practical interest to study the relation between these concepts and the conventional reliability concepts of mean time to failure and scatter range, because of the advantage of a structural reliability analysis based on order statistics rather than on conventional statistics.

II. Distribution Functions of Physical Significance

In both materials tests and life tests of structures the number of test replications that is technically and economically feasible precludes the derivation of the distribution function of fatigue lives from observations by methods of statistical inference. It is therefore necessary to devise, on the basis of relevant physical arguments, probabilistic models which produce distribution functions that are germane to the physical phenomenon, so that their use can be justified in the extrapolation from small numbers of test results to probability levels as far removed from the range of test results as the levels that are significant in reliability analysis. However, it should be realized that only very simple physical arguments can

be translated into probability statements which result in a characteristic distribution function.

Thus, for instance, the argument, based on observations of step-like crack propagation under cyclic loading, that each new crack element can be considered a partial static rupture, independent of preceding ruptures, leads to the conclusion that cracking is a Poisson process[2]. The probability of failure associated with the exceedance of a finite number r of elementary cracks is thus given as the sum of a finite number of Poisson terms

$$P\{x \geq r\} = \sum_{o}^{r} p(x) = e^{-L/T} \sum_{o}^{r} \frac{1}{x!} (t/T)^{x} = P(t) \qquad (2.1)$$

where T is the return period of the elementary crack process of probability $p = T^{-1}$, and x the number of occurrences in a given time interval t. The time to failure as a function of r (mortality function) is obtained by differentiation with respect to t of $P^*=[1-P(t)]$

$$dP^*/dt = \frac{1}{r!T} (t/T)^{r} e^{-t/T} \qquad (2.2)$$

which, with $x = t$, $\alpha = r + 1$, $\lambda = T^{-1}$ and $\Gamma(\alpha) = (\alpha-1)!=r!$ is the Gamma function

$$p(x) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)} x^{(\alpha-1)} e^{-\lambda x} \qquad (2.3)$$

Alternatively, if it is assumed that an elementary crack length ε is exponentially distributed, the momentary crack length $\ell = \sum_{1}^{k} \varepsilon_i$ follows a Gamma distribution[3] for the crack length $x = \ell$ with $\alpha = k$ and $\lambda = 1/\bar{\varepsilon}$, where $\bar{\varepsilon}$ is the mean elementary crack length, since this is the distribution of the sum of k random variables, each of exponential distribution. Since time to failure can be associated with the time to attain a certain crack length, since this is the distribution of the sum of k random variables, each of exponential distribution. Since time to failure can be associated with the time to attain a certain crack length, both physical arguments lead to Gamma distributions for the time to failure.

The physical assumption of an increase of the rate of cracking with increasing number N of load applications and thus with increasing time t can be translated into the probabilistic requirement of a risk function or "hazard rate", increasing with increasing t. The simplest form of this function satisfying such assumption

$$h(t) = \alpha \left(\frac{t}{v}\right)^{\alpha-1} \qquad \alpha > 1 \tag{2.4}$$

where v is a measure of the central tendency of t leads to the extreme value distribution

$$P^*(x) = e^{-(x/v)^{\alpha}} \tag{2.5}$$

with x = t; the scale parameter α is an inverse measure of the scatter of x or t, which implies that the more rapidly the increase of the risk function the narrower the associated scatter of lives. Eq.(2.5) represents the Third asymptotic distribution function of the shortest lives[4] in large samples of well-behaved statistical populations of unknown distribution, also known as the Weibull distribution.

Since experiments have shown that a certain "incubation period" t_o, the minimum life[5], is needed to start the process of crack propagation, a more appropriate formulation of the hazard rate

$$h(N) = \alpha \left(\frac{t-t_o}{v-t_o}\right)^{\alpha-1} \qquad \alpha > 1 \tag{2.6}$$

leads to an extreme value distribution with lower limit

$$P^*(x) = e^{-\left(\frac{x-x_o}{v-x_o}\right)^{\alpha}} \tag{2.7}$$

with $x = t$ and $x_o = t_o$.

The wide use of the Logarithmic Normal distribution for fatigue lives is, in general, based on arguments of expediency (use of tables and methods of estimation developed for Normal distribution), rather than physical or probabilistic reasoning, although an argument that the damaging effect of each load application is proportional to the total damage produced by the preceding load cycles can be used to justify the logarithmic normal distribution of fatigue lives[6]. The application of the central limit theorem to a product of statistical variables provides further justification, although, physically, the above arguments do not carry conviction.

The hazard functions of the three distribution functions are quite dissimilar: for the Gamma distribution it increases asymptotically toward a constant value associated with chance failures, for the Weibull distribution it increases monotonicaly, for the logarithmic normal it first increases at decreasing rate, but later decreases very slowly towards zero. However, an experimental demonstration of the tendencies of these functions is just as impractical as a demonstration of the forms of the associated reliability or frequency functions themselves (Fig. 1) since the significant differences in their trend only appear in the central range of lives.

Certain physical assumptions arising from considerations of structural failure phenomena lead to composite distribution functions of various types. Thus, for instance, the assumption that fatigue failure of a structure subject to variable load intensities is essentially an ultimate load (chance) failure of the structure that has been progressively weakened by propagating fatigue cracks[7] may be expressed by replacing the constant "return period" of ultimate load failure $v = 1/P_f$ in the exponential distribution characteristic for chance failures

$$P^*(x) = e^{-(x/v)} \qquad (2.8)$$

by a decreasing function $v(x)$ of x reflecting the progressive deterioration by fatigue of the initial ultimate load resistance R. The probability of ultimate load failure of the undamaged structure is obtained from the distribution function $P(\nu)$ of the safety factor $\nu = R/S$, where R and S denote respectively the resistance and the load considered as statistical variables[8], at $\nu = 1$, or $P(\nu) = P(1) = P_f$. Various assumptions could be made concerning the form of the function $v(x)$. Of these the function

$$v(x) = v_o[1-(\frac{x}{u})^n] > o \qquad \text{with } n > 1 \qquad (2.9)$$

seems best suited to reproduce actual conditions. The parameter u denotes the time x at which the resistance R has been completely destroyed by fatigue cracking. Obviously the time u is a function of the operational load spectrum that causes fatigue, of the fatigue resistance of the structure which, in turn, depends on the fatigue performance of the material and on details of the structural design, and on the maintenance, repair or replacement program as a result of which the time u is periodically extended. Moreover u is not a constant but a statistical variable, since it represents the fatigue life of the structure subject to the normal operational load spectrum alone, independently of (extreme) load spectra that are likely to produce actual failure (thunderstorm gusts, low-level operations, etc.). In Eq. (2.9) this variable is replaced by its expected (mean) value. The power $n > 1$ reproduces the observation that the effect of fatigue deterioration of the static resistance proceeds at first very slowly but accelerates sharply as x approaches u. Introducing Eq. (2.9) into (2.8) the distribution function is obtained

$$P^*(x) = e^{-(x/v_o)\,[1-(\frac{x}{u})^n]^{-1}} \qquad (2.10)$$

The risk function associated with Eq. (2.10) follows from

$$h(x) = -\frac{d}{dx}\ln P^* = \frac{d}{dx}\frac{(x/v_o)}{[1-(\frac{x}{u})^n]} = \frac{1+(n-1)(\frac{x}{u})^n}{[1-(\frac{x}{u})^n]^{-2}} \tag{2.11}$$

Developing the exponent of Eq. (2.10) into a power series

$$(\frac{x}{v_o})[1-(\frac{x}{u})^n]^{-1} = (\frac{x}{v_o})[1+(\frac{x}{u})^n+(\frac{x}{u})^{2n}+\ldots]; \quad (\frac{x}{u})^2 < 1 \tag{2.12}$$

which for values $n > 1$ converges rapidly in the region of practical significance $(\frac{x}{u}) < 0.5$, Eq. (2.10) can be written in the form

$$P^*(x) = e^{-(\frac{x}{v_o})} \cdot e^{-\frac{1}{\varphi}(\frac{x}{u})^{n+1}} \cdot e^{-\frac{1}{\varphi}(\frac{x}{u})^{2n+1}} \cdots$$

$$= \pi_m e^{-\frac{1}{\varphi}(\frac{x}{u})^{mn+1}} = \pi_m P^*_m(x) \tag{2.13}$$

for $(x/u) < 1$ and $m = 0,1,2,\ldots,$ where the "fatigue ratio" $\varphi = (v_o/u)$ relates the return period of ultimate load failure to the expected time to complete loss of resistance caused by fatigue.

The function $P^*(x)$ in Eq. (2.13) is a product of extreme value distributions $P^*_m(x)$ of the type of Eq. (2.5) with $\alpha_m = mn+1$. Since α_m is inversely proportional to the standard deviation $\sigma_m(\log_{10}t) = \delta_m$

$$\delta_m = \pi/2.303\alpha_m\sqrt{6} \tag{2.14}$$

it follows that the dispersion of the functions $P_m(x)$ decreases with increasing m.

The risk function associated with Eq. (2.13)

$$h(x) = -\frac{d}{dx}\ell n P^*(x) = \frac{d}{dx}\varphi^{-1}\sum_m\left(\frac{x}{u}\right)^{mn+1} = \frac{1}{v_o}\sum_m (mn+1)\left(\frac{x}{u}\right)^{mn} \tag{2.15}$$

for $(\frac{x}{u}) < 1$, is an increasing function of x, resulting from linear superposition of processes of progressive damage. For large values of n the series converges fast enough to justify the approximation

$$h(x) \doteq \frac{1}{v_o}\left[1 + (n+1)\left(\frac{x}{u}\right)^n\right] = \frac{1}{v_o} + (n+1)\frac{1}{u_1}\left(\frac{x}{u_1}\right)^n \tag{2.16}$$

which represents the simple superposition of a chance risk and a progressive damage risk function with expected life $u_1 = u\sqrt[n+1]{\varphi} > u$. $P^*(x)$ is expressed by the first two terms of Eq. (2.13):

$$P^*(x) = e^{-\frac{1}{\varphi}(x/u)} \cdot e^{-\frac{1}{\varphi}(x/u)^{n+1}} \tag{2.17}$$

Distribution functions of the general type of Eq. (2.13)

$$P^*(x) = \pi_m P^*_m(x) \tag{2.18}$$

arise in the reliability analysis of multicomponent structures or of structures subject to a multitude of independent effects. The number m of product terms may represent the number of components surviving at time x or the number of effects acting at time x on the surviving structure.

III. Application of Order Statistics

Certain order statistics ofseveral distribution functions are evaluated and compared with the expectation that the results will provide a basis for comparison of tests and ovservations on structures and structural parts which is more effective than conventional statistical methods, as well as a basis for the establishment of a rational and administrable procedure of reliability demonstration.

It is instructive to evaluate first the order statistics for the exponential distribution used to represent chance failures.

The expected value of the m-th order statistics of a sample of size n from an exponential population specified by Eq. (2.8)

$$P(x) = 1 - P^*(x) = 1 - e^{-(x/v)} \tag{3.1}$$

and

$$p(x) = \frac{1}{v} e^{-(x/v)} \tag{3.2}$$

where v is the expected mean time to failure

$$E(x_m)_n = v \sum_{i=1}^{m} \frac{1}{n-i+1} \tag{3.3}$$

Hence

$$E(x_1) = \frac{1}{n} v; \quad E(x_2) = \left[\frac{1}{n} + \frac{1}{n-1} \right] v \tag{3.4}$$

and

$$E(x_2 - x_1) = E(x_2) - E(x_1) = \frac{1}{n-1} v \tag{3.5}$$

When the sample size is moderate or large, the expected times to first and second failure are quite short, much shorter, as will be seen, than these times for any of the considered distributions of fatigue life. This is an important fact when design criteria for ultimate load failure are compared with design criteria for fatigue. If avoidance of even a single catastrophic failure is a design requirement, expected

times to first failure must replace the mean times to failure as significant design criteria. In the case of ultimate load failure the requirement of an expected time to first failure of $x_1 = 100{,}000$ hrs. in a fleet of $n = 1000$ units would imply a design mean time to failure for the fleet of $v = 10^8$ hrs., producing a reliability function $P^*(x) = \exp(-x/10^8)$ with a probability of failure of $P_F = 1 - e^{-10^{-3}}$ $\sim 10^{-3}$ at $x = x_1 = 10^5$. The expected interval between the first two consecutive failures is about 10^5 hrs.

For a distribution function $P(x)$ and frequency function $p(x)$ the probability that no failure will occur in the interval between o and x and one of a sample of size n will fail in the interval between x and $(x+\Delta x)$ is given by the expression

$$p(X_1) = n[1-P(x)]^{n-1} \, p(x)\,\Delta x \tag{3.6}$$

The expected time to the first failure is the expectation of Eq. (3.6), or

$$EX_1 = n \int_{-\infty}^{+\infty} x[1-P(x)]^{n-1} p(x)\,dx \tag{3.7}$$

Similarly, the time to the second failure

$$EX_2 = n(n-1) \int_{-\infty}^{+\infty} xP(x)[1-P(x)]^{n-2} p(x)\,dx \tag{3.8}$$

The expected value of the interval between the first and second failures

$$E(X_2 - X_1) = EX_2 - EX_1 \tag{3.9}$$

The variance of X, is obtained from

$$\text{Var } X_1 = EX_1^{\;2} - (EX_1)^2 \tag{3.10}$$

where

$$EX_1^{\;2} = n \int_{-\infty}^{+\infty} x^2 \, [1-P(x)]^{n-1} p(x)\,dx \tag{3.11}$$

The variance of X_2

$$\text{Var } X_2 = EX_2^{\ 2} - (EX_2)^2 \tag{3.12}$$

where

$$EX_2^{\ 2} = n(n-1)\int_{-\infty}^{+\infty} x^2 P(x)[1-P(x)]^{n-2} p(x)dx \tag{3.13}$$

The coefficient of variation of $P(x)$

$$V_x = \sqrt{\text{Var}}\ X/EX \tag{3.14}$$

For the Third Asymptotic Extremal Distribution[9] according to Eq. (2.5) with

$$P(x) = 1 - P^*(x) = 1 - e^{-(x/v)^\alpha} \tag{3.15}$$

and

$$p(x) = (\alpha/v)(x/v)^{\alpha-1} e^{-(x/v)^\alpha} \tag{3.16}$$

evaluation of Eqs. (3.7) to (3.13) produces the expressions:

$$EX_1 = vn^{-1/\alpha}\Gamma(1+1/\alpha) \tag{3.17}$$

$$EX_1^{\ 2} = v^2 n^{-2/\alpha}\Gamma(1+2/\alpha) \tag{3.18}$$

$$EX_2 = v\Gamma(1+1/\alpha)[n(n-1)^{-1/\alpha} - (n-1)n^{-1/\alpha}] \tag{3.19}$$

$$EX_2^2 = v^2\Gamma(1+2/\alpha)[n(n-1)^{-2/\alpha} - (n-1)n^{-2/\alpha}] \tag{3.20}$$

while

$$EX = v\Gamma(1+1/\alpha) \tag{3.21}$$

and

$$\text{Var } X = [\Gamma(1+2/\alpha) - \Gamma^2(1+1/\alpha)]v^2 \tag{3.22}$$

Hence

$$EX_1 = n^{-1/\alpha} EX \tag{3.17a}$$

$$EX_1{}^2 = n^{-2/\alpha} EX^2 \tag{3.18a}$$

$$\mathrm{Var}\ X_1 = n^{-2/\alpha} \mathrm{Var}\ X \tag{3.23}$$

and therefore

$$\sigma(X_1) = n^{-1/\alpha} \sigma(X) \tag{3.24}$$

The coefficient of variation

$$v_{x_1} = \sigma(X_1)/EX_1 = \sigma(X)/EX \tag{3.25}$$

is therefore identical with that of the parent population.

Eqs. (3.17a) to (3.24) can also be used to predict the parameters of the parent population from observations of the times to first failure in samples of size n

$$EX = n^{1/\alpha} EX_1 \tag{3.17b}$$

and

$$\sigma(X) = n^{1/\alpha} \sigma(X_1) \tag{3.24a}$$

If Eq(2.7) is considered with the lower bound (minimum life) specified as a fraction of the characteristic life $x_o = \beta v$ so that

$$P^*(x) = e^{-\left[\frac{x-\beta v}{v(1-\beta)}\right]^\alpha} \tag{2.7a}$$

the transformation

$$X = Y + \beta v \tag{3.26}$$

reestablishes the form of Eq. (3.15) with respect to the variable Y

$$P(y) = 1-P^*(y) = 1-e^{-(y/w)^{\alpha}} \tag{3.27}$$

where $w = v(1-\beta)$. Hence Eqs. (3.17) to (3.20) are valid with respect to the variable Y and the following expressions can be written for the order statistics of Eq. (3.27):

$$EY_1 = wn^{-1/\alpha}\Gamma(1+1/\alpha) \tag{3.28}$$

or, since

$$Y_k = X_k-\beta v \tag{3.26a}$$

the expectation

$$EX_1 = \beta v+v(1-\beta)n^{-1/\alpha}\Gamma(1+1/\alpha) = \beta v+(1-\beta)EX_1^{(o)} \tag{3.29}$$

where $EX_1^{(o)}$ is the value of EX_1 for $x_o = 0$ given by Eq. (3.17). Similarily

$$EX_2 = \beta v + (1-\beta)EX_2^{(o)} \tag{3.30}$$

where EX_2 is given by Eq. (3.19). The expected value of the interval (X_2-X_1) according to Eqs. (3.9), (3.29) and (3.30)

$$E\ (X_2-X_1) = (1-\beta)\left[EX_2^{(o)} - EX_1^{(o)}\right] \tag{3.31}$$

while

$$EX = \beta v + v(1-\beta)\ \Gamma(1+1/\alpha) \tag{3.32}$$

Because of Eq. (3.26a)

$$\mathrm{Var}\ X_k = \mathrm{Var}\ Y_k \tag{3.33}$$

and

$$\mathrm{Var}\ (X_{k+1}-X_k) \quad = \quad \mathrm{Var}\ (Y_{k+1}-Y_k) \tag{3.34}$$

In Fig. 2 the ratios EX_1/EX, $E(X_2-X_1)/EX$, $\mathrm{Var}\ X_1/(EX)^2$ and $\mathrm{Var}\ (X_2-X_1)/(EX)^2$ are plotted as functions of the coefficient of variation V_x of the distribution for $x_o = 0$ and $X_o = 0.1\ v$ (β=0.1) and sample sizes n = 3, 20, 50, 200 and 1,000. The effect of the existence of a minimum life is quite pronounced although the assumed value of β=0.1 is moderate. That a value of this order of magnitude can be expected even within the range of short and moderate fatigue lives is suggested by Fig. 3 which shows values of $x_o = N_o$ evaluated from fatigue tests on aluminum specimens [10] and fitted by a straight line, compared with the relation $N_o = 0.1v \sim 0.11\bar{N}$, assuming that for $3 < \alpha < 4$ the relation $\bar{N} = v\Gamma(1+1/\alpha) \sim 0.9v$ is a good enough approximation. If the reliability analysis is performed under the convenient assumption $x_o = 0(\beta=0)$, the actual existence of a minimum life provides a significant additional margin of safety. If the load spectrum is specified in terms of the percentages p_i of time during which the structure is subject to load amplitudes of certain (constant) intensities S_i, each of which, if acting alone, would produce failure at a specified mean time EX_i, and if the distributions of the times X_i are assumed extremal with characteristic values v_i, $x_{oi} = 0$ and the same coefficient of variation ($\alpha_i = \alpha$), the reliability function under the assumption of no interaction between load intensities is of the form

$$P^*(x) = \pi_i \exp\left(-\frac{p_i x}{v_i}\right)^{\alpha} = \exp\left[-\Sigma\left(\frac{p_i x}{v_i}\right)^{\alpha}\right] = \exp\left(-\frac{x}{w}\right)^{\alpha} \tag{3.35}$$

where $w^{-\alpha} = \sum_i \left(\frac{p_i}{v_i}\right)^{\alpha}$; w is the characteristic time to failure under the combined loading. For $\alpha = 1$. Eq. (3.35) implies linear damage accumulation. In the case of interaction between load intensities the values

v_i are reduced by interaction factors $\omega_i > 1$ the values of which increase with decreasing load intensity. In this case the characteristic value w is obtained from the relation

$$w^{-\alpha} = \sum_i \left(\frac{p_i \omega_i}{v_i}\right)^{\alpha} \tag{3.36}$$

As a result of this interaction the damaging effect of low load intensities is considerably magnified. The expected time to first failure in a sample of size n as well as the other order statistics can be obtained from Eqs. (3.17) to (3.20) or from the diagrams in Fig. 2.

Eq. (3.7) cannot be evaluated in closed form for the reliability function Eq. (2.17). Numerical evaluation within the range of validity of its approximation of Eq. (2.10) $(x_1/u) < 1.0$ for the exponents $\alpha = n + 1 = 2, 3, 4$ the parameters $\varphi =$ 10,100 and 1,000 and the sample size n = 20, 50, 200 and 1,000 produces the values EX_1 in terms of u presented in Table 1:

Table 1.

n \ φ	10	100	1000	α
	0.362			2
20	0.381			3
	0.392			4
	0.177	0.836		2
50	0.186	0.820		3
	0.190	0.811		4
	0.045	0.361		2
200	0.049	0.380		3
	0.049	0.392		4
	0.007	0.088	0.568	2
1000	0.007	0.096	0.582	3
	0.007	0.099	0.591	4

The expected time to the first chance failure for the reliability function Eq. (2.8) with $v = \varphi u$ according to the first Eq. (3.4) is

$$EX_{1C} = \frac{1}{n} v = \frac{\varphi}{n} u \qquad (3.37)$$

The values of the ratio (EX_1/EX_{1C}) are presented in Fig. 4 as a function of EX_{1C}. It can be seen that these values are practically independent of n and of α. For values $EX_{1C} < 0.2u$ the ratio is sufficiently close to unity to justify the conclusion that under the assumptions resulting in the reliability function Eq. (2.17) fatigue reduces the expected time to first chance failure only when $\varphi < 0.2n$ or $u < 5v/n$.

IV. Fatigue Sensitivity

The reliability estimate of a structure is strongly affected by its fatigue sensitivity. For fatigue insensitive structures the reliability estimate depends on adequate statistical correlation of the carrying capacity of the structure with a spectrum of extreme loading conditions, such as a thunderstorm or an extremal maneuver load spectrum. The statistical variability of the initial carrying capacity has recently been established on the basis of evaluation of industrial test data on full-scale structural elements[11] and spectra of extreme loading conditions as well as their frequency ratio within the total operational conditions can be reasonably well constructed on the basis of extrapolation from flight records obtained under extremal conditions. The reliability function of the structure for ultimate load failure can therefore be derived on the basis of the assumed chance character of this failure; its return period $v = P_F^{-1}$ is obtained by evaluation of the probability of failure of the structure from the correlation of carrying capacity and load-spectrum.[12]

When the structure is fatigue sensitive the dependence of the momentary carrying capacity of the structure on its operational history destroys the possibility of this simple correlation. The ob-

servation that under variable load intensities fatigue failures in tests as well as in operation occur at the highest load intensities applied, justifies the assumption that the chance character of the failure process can be retained; however, the reduction of the carrying capacity as a function of the load-history must be introduced. This leads to a reliability function of the type of Eq. (2.10) the actual use of which is severely limited by the analytical difficulties of its use. The proposed approximation Eq. (2.17) makes its use over a limited range possible.

With respect to this reliability function the range of fatigue insensitivity has been delimited by the values of the expected time to first chance failure $0 < EX_{1C} < 0.2u$.

Thus, for a fleet size of $n = 50$ the condition for a fatigue insensitive design is $u > 0.1v$ while for $n = 1000$ $u > 0.005v$. Introducing as an example, $EX_{1C} = 2x10^5$ hrs. for $n = 1000$ as an admissible design value, the expected time to chance failure $EX_C = v = 2x10^8$ hrs; a fatigue-insensitive design therefore requires $u > 10^6$ hrs. For the same value EX_{1C} in a smaller fleet of $n = 50$ for which $v = 10^7$ hrs, the requirement for fatigue insensitive-design is again $u > 10^6$ hrs. Assuming therefore that the design limit load, considered as the highest load intensity of the fatigue spectrum, is determined by the requirement $u > 10^6$ hrs, independently of fleet size, the design ultimate load intensity should be the higher the larger the fleet size. This requirement obviously interferes with the constant ratio between ultimate and limit load on which conventional design is based.

If, for the sake of more convenient analytical manipulation, the reliability function of a fatigue sensitive structure is approximated by the extremal distribution Eq. (2.5) alone, while Eq. (2.8) is the reliability function of the undamaged structure for ultimate load failure, the range of fatigue insensitivity is delimited by the condition that the expected time to first ultimate load failure EX_{1C} be

smaller than the time to first failure EX_{1F} of the fatigue sensitive structure. Considering Eqs. (3.17a) and (3.4) the ratio between EX_{1F} and EX_{1C} considered as inverse fatigue sensitivity factor

$$f_1^{-1} = EX_{1F}/EX_{1C} = n^{-1/\alpha} EX_F/vn^{-1} = (EX_F/EX_C)n^{(1-1/\alpha)} \tag{4.1}$$

since $v = EX_C$. The values of $n^{(1-1/\alpha)}$ are given in Table 2 for n = 3, 20, 50, 200, 1000 and α = 2, 3, 4.

Table 2

α \ n	3	20	50	200	1000
2	1.7	4.5	7.1	14.2	31.6
3	2.1	7.7	13.6	34.2	100
4	2.3	9.4	18.9	53.8	179

These values are the ratios between the design values (EX_C/EX_F) necessary to ensure that $EX_{1F} \geqq EX_{1C}$, a condition that might be used to designate a structure as fatigue-insensitive ($f_1 < 1$). When the ratio is higher the factor $f_1 > 1$ and the structure becomes fatigue-sensitive since the expected time to the first fatigue failure is shorter than that to the first chance failure. For n = 1000 at α = 4 and for n = 50 at α between 2 and 3 these ratios are fairly close to those obtained from Eq. (2.17) which, however, are practically independent of α. Nevertheless the fact that the right order of magnitude of these ratios in the range of the shape parameter $2 < \alpha < 4$ associated with the range of significant values of the scatter of fatigue life $0.14 < \sigma(\log X_F) < 0.19$ is obtained from Eq. (4.1) might justify the use of this equation for the illustration of the significance of the concept of fatigue sensitivity in an integrated design procedure.

If it can be assumed that an estimate of EX_F under a cyclic load spectrum $p_i(S_i)$ is obtainable from a modified linear damage accumulation rule

$$\sum_i \frac{p_i \, \bar{m} \, EX_F}{\bar{m} EX_{iF}} = k \tag{4.2}$$

where EX_{iF} is the expected time to failure in hours at the constant load amplitude S_i, taken either from an average S-N diagram by converting the mean number of cycles $\bar{N}_i = \bar{m}EX_{iF}$ into hours with the aid of the expected number $\bar{m}$ of cycles per hour, in which case $k < 1$, or from an S-N diagram corrected for stress-interaction[13] in which case $k = 1$, so that

$$EX_F = k \left[\sum_i \frac{p_i}{EX_{iF}} \right]^{-1} \tag{4.3}$$

the design ratio

$$f = EX_C / EX_F = \left[\sum_i \frac{p_i}{EX_{iF}} \right] / \; kP_F \tag{4.4}$$

where P_F is the probability of ultimate load failure per hour of flight. The values p_i, EX_{iF} and k are determined by the fatigue load spectrum and the fatigue behavior of the material in the structure, while P_F combines the effects of ultimate load spectrum and ultimate carrying capacity of the structure. The "design ratio" f thus reflects the combined principal design parameters. Since Eq. (4.1) can be written in the form

$$f = f_1 n^{(1 - 1/\alpha)} \tag{4.5}$$

it takes the form

$$f = n^{(1 - 1/\alpha)} \tag{4.6}$$

for the critical value $f_1 = 1$. The structure is fatigue sensitive when $f_1 > 1$ and therefore f larger than the values tabulated in Table 2 and fatigue insensitive when $f_1 < 1$ so that f is smaller than these values.

When a structure has been designed for ultimate load with a probability of failure for a single load application p_F and with an associated probability of failure per hour $P_F = mp_F = [EX_C]^{-1}$, where m is the number of load application per hour in the range of the ultimate load spectrum, Eqs. (4.4) and (4.5) can be used to determine its fatigue sensitivity factor f_1 when the fatigue load spectrum and the fatigue performance of the material in the structure subject to this spectrum are known; conversely, the maximum intensity of the fatigue-load spectrum can be determined for which the condition $f_1 = 1$ is satisfied. Thus, for instance for a fleet size n of structures designed for ultimate load with probability p_F the condition of fatigue-insensitivity $f_1 \leqq 1$ takes the form $f \leqq n^{(1-1/\alpha)}$ or

$$\sum_i \frac{p_i}{EX_{iF}} \leqq n^{(1-1/\alpha)} kmp_F \tag{4.7}$$

Assuming an average of 20 gusts per mile of flight in thunderstorm turbulence all of which are of an intensity that can be considered to fall within the extreme range from which the ultimate load intensity is extrapolated, and assuming further that flight through thunderstorm turbulence makes up about 0.1 percent of operational flight time[14] with average velocity through turbulence of 400 miles, the expected number of applications of loads belonging to the ultimate load spectrum is m = 8 gusts per hour of (total) flight time. For $n = 1000$, $\alpha = 4$, $p_F = 10^{-8}$ and $k \sim 0.5$ Eq. (4.7) expresses the condition

$$\sum_i \frac{p_i}{EX_{iF}} \leqq 179 \times 0.5 \times 8 \times 10^{-8} = 7.15 \times 10^{-6}$$

For a fatigue load spectrum of five load levels: S_5 with $p_4 = 10^{-4}$ and $EX_{SF} = N_5/\bar{m}$, S_4 with $p_4 = 10^{-3}$ and $EX_{4F} = 10N_5/\bar{m}$, S_3 with $p_3 = 10^{-2}$ and $EX_3 = 100N_5/\bar{m}$, S_2 with $p_2 = 10^{-1}$ and $EX_{2F} = 1000N_5/\bar{m}$

and S_1 with $p_1 = 0.889$ and $EX_{1F} = 10^4 N_5/\bar{m}$ the left side of Eq. (4.7) becomes

$$\sum_i \frac{p_i}{EX_{iF}} = 4.889 \times 10^{-4} \bar{m} N_5^{-1}$$

The number $\bar{m}$ can be estimated from the assumption of an average of 0.1 gusts that produce fatigue damage per mile of normal flight at 400 miles per hour or $\bar{m} = 40$ such gusts per hour. Hence fatigue insensitivity of the design requires that the number of cycles to failure at the highest load level S_5 should be $N_5 \gtrsim 2700$ cycles or $EX_{5F} = 68$ hrs. In a total estimated life of 25,000 hrs. this load intensity is actually applied during 2.5 hours or an expected number of 100 times; S_5 is therefore only somewhat lower than the "limit load" defined by an expected frequency of occurence of about 10, if the structure is to be fatigue insensitive with respect to the first failure. This conclusion would provide some theoretical support to the rule of thumb used in industry[15] that a structure or structural part will be fatigue insensitive in operation if it can support about 3000 repetitions of the limit load.

The fatigue sensitivity design factor f is a function of n according to Eq. (4.5). Therefore the fatigue sensitivity factor f_1 also changes with fleet size. For the considered design conditions the ratio which is $f_1 = 1$ for $n = 1000$ is increased for $n = 50$ by 100/13.6 to $f_1 = 7.3$ so that $EX_{1F} = 0.136 EX_C$: the smaller the fleet the more fatigue sensitive it is with respect to first failure. This only means that the smaller the fleet the more likely it is that the first failure is not an ultimate load failure but a fatigue failure. If fatigue sensitivity were defined with respect to the ratio of expected times $f = 1.0$ rather than with respect to f_1, the fatigue sensitivity $f_1 = n^{-(1-1/\alpha)} < 0$: the structure would always be highly fatigue-insensitive.

For this condition, however, the highest load level S_5 of the considered spectrum and n = 1000 would be associated with $N_1 \gtrsim 5 \times 10^5$ and thus be far below the limit-load.

Definition of fatigue sensitivity by the ratio of the risk functions associated with Eqs. (2.5) and (2.8)[16]

$$f' = \frac{r_F}{r_U} = \alpha \frac{v}{v_F} \left(\frac{t}{v_F}\right)^{\alpha-1} \tag{4.8}$$

since

$$r_F = \frac{\alpha}{v_F} \left(\frac{t}{v_F}\right)^{\alpha-1} \quad \text{and} \quad r_U = v^{-1} \tag{4.9}$$

can be related to the ratio f_1 with the aid of Eqs. (3.21) and (3.17)a

$$f' = \alpha f \left[\Gamma(1 + 1/\alpha)\right]^{\alpha} \left(\frac{t}{EX_F}\right)^{\alpha-1} \tag{4.10}$$

Introducing $t = EX_{C1} = EX_C/n$

$$f' = \alpha f^{\alpha} \left[\Gamma(1 + 1/\alpha)\right]^{\alpha} n^{-(\alpha-1)} \tag{4.11}$$

and therefore

$$f' = \alpha f_1^{\alpha} \left[\Gamma(1 + 1/\alpha)\right]^{\alpha} \tag{4.12}$$

Hence for $f_1 = 1.0$

$$f' = \alpha \left[\Gamma(1 + 1/\alpha)\right]^{\alpha} > 1 \tag{4.13}$$

The time t* at which the risks r_F and r_U are equal which has been proposed as a safe life of operation[17] is obtained from Eq. (4.10) with $f' = 1$

$$t^* = EX_F \sqrt[\alpha-1]{(\alpha f)^{-1} [\Gamma(1+1/\alpha)]^{-\alpha}} = EX_C f^{-\alpha/\alpha-1} \sqrt[\alpha-1]{\alpha^{-1} [\Gamma(1+1/\alpha)]^{-\alpha}} \tag{4.14}$$

Since

$$f^{-\alpha/\alpha-1} = \frac{1}{n} f_1^{-\alpha/\alpha-1}$$

the expected time to equal risk of failure

$$t^* = EX_{1C} f_1^{-\alpha/\alpha-1} \sqrt[\alpha-1]{\alpha^{-1}[\Gamma(1+1/\alpha)]^{-\alpha}} \tag{4.15}$$

For a design with $f_1 = 1$ this time $t^* < EX_{1C} = EX_{1F}$; for $2 < \alpha < 4$ the ratio $0.63 < (t^*/EX_{1F}) < 0.73$. Thus the fatigue sensitivity criterion $f_1 = 1$ produces a time to first failure which differs only slightly from the time at which the failure risks are equal. This time $t^* \sim 2/3\ EX_{1F}$ can thus be considered as a fair estimate of safe life.

Obviously the consideration of f_1 as the constant ratio $f_1 = EX_{1C}/EX_{1F}$ is only an approximation to the more rigorous approach in which X_{1C} and X_{1F} are introduced as random variables so that the fatigue sensitivity factor f_1 as a quotient of two random variables

$$f_1(X) = X_{1C}/X_{1F} \tag{4.16}$$

is itself a random variable. It has been assumed, so far that the expectation Ef_1 is closely enough approximated by the ratio of the expectations of X_{1C} and X_{1F}. But unless the distribution of $f_1(x)$ is known no probability statement can be associated with the criterion of fatigue-insensitivity $f_1(x) \leqq 1$.

The density function of the time to the first failure $p(X_1)$ is given by Eq. (3.6) for any given distribution of times to failure $P(x)$. Thus, for instance, for the distribution function Eq. (2.5)

$$p(X_{1F}) = p_{1F}(x) = n \frac{\alpha}{v_F}\left(\frac{x}{v_F}\right)^{\alpha-1} e^{-n(x/v_F)^{\alpha}} \tag{4.17}$$

which, for $\alpha = 1$, becomes

$$p(X_{1C}) = p_{1C}(x) = n \frac{1}{v_C} e^{-n(x/v_C)} \tag{4.18}$$

The distribution of the quotient $f_1 = (X_{1C}/X_{1F})$ is therefore[17]

$$P(f_1) = \int_0^{\infty} p_{1C}(xf_1)p_{1F}(x)dx = \tag{4.19}$$

$$= 1 - \int_0^{\infty} n\alpha \left(\frac{x}{v_F}\right)^{\alpha-1} \exp\left\{- n\left[\frac{v_F}{v_C} f_1 \left(\frac{x}{v_F}\right) + \left(\frac{x}{v_F}\right)^{\alpha}\right]\right\} d\left(\frac{x}{v_F}\right)$$

and

$$p(f_1) = \int_0^{\infty} n^2 \frac{v_F}{v_C}\alpha \left(\frac{x}{v_F}\right)^{\alpha} \exp\left\{- n\left[\frac{v_F}{v_C} f_1 \left(\frac{x}{v_F}\right) + \left(\frac{x}{v_F}\right)^{\alpha}\right]\right\} d\left(\frac{x}{v_F}\right) \tag{4.20}$$

The form of the distribution and of the density function can only be evaluated numerically. This has been done for $\alpha = 3,4$ and $n = 3$, 20, 50, 200 and 1000 and the distribution functions $P(f_1)$ are plotted in Fig. 5 on the scale $f_1(v_F/v_C)$. However the expectation $E\{f_1\}$ and the variance $\sigma_{F1}^{\ 2}$ can be obtained in closed form. Thus after some manipulation

$$E\{f_1\} = \int_0^{\infty} f_1 . p(f_1)df_1 = \frac{v}{v_F} \ n^{-(1-1/\alpha)} . \Gamma(1-1/\alpha) \tag{4.21}$$

Considering that $v = EX_C$ and $v_F = EX_F[\Gamma(1+1/\alpha)]^{-1}$ this expression can be written in the form

$$E\{f_1\} = f\ n^{-(1-1/\alpha)}\Gamma(1-1/\alpha).\Gamma(1+1/\alpha) \tag{4.22}$$

of which Eq. (4.5) is an approximation.

The product of the gamma functions in Eq. (4.22) for $\alpha = 2$, 3 and 4 is, respectively (1.77x0.88) = 1.55, (1.35x0.89) = 1.20 and (1.225x0.91) = 1.10; the approximate values of Table 2 are the closer to the correct values of Eq. (4.22) the larger the parameter α. For $n = 1000$ and $\alpha = 4$ used in the illustration of the procedure the correct ratio is (179/110) = 163, which produces the correct value of $N_5 \geqq 3000$ cycles or $EX_{5F} = 75$ hrs.

The variance

$$\sigma_{F1}^{\ 2} = E\{f_1^{\ 2}\} - [E\{f_1\}]^2 \tag{4.23}$$

Evaluating

$$E\{f_1^{\ 2}\} = \int_0^\infty f_1^{\ 2} p(f_1)\, df_1 \;=\; \left[\frac{v}{v_F}\ n^{-(1-1/\alpha)}\right]^2 \Gamma(3).\Gamma(1-2/\alpha) \tag{4.24}$$

valid for $\alpha > 2$. The variance

$$\sigma_{f1}^{\ 2} = \left[\frac{v}{v_F}\ n^{-(1-1/\alpha)}\right]^2 \{2\Gamma(1-2/\alpha) - [\Gamma(1-1/\alpha)]^2\}$$

$$= [E\{f_1\}]^2 \left\{ \frac{2\Gamma(1-2/\alpha)}{[\Gamma(1-1/\alpha)]^2} - 1\right\} \tag{4.25}$$

The coefficient of variation of F_1 is therefore

$$\nu_{f1} = \left\{ \frac{2\Gamma(1-2/\alpha)}{[\Gamma(1-1/\alpha)]^2} - 1\right\}^{1/2} \quad \text{for } \alpha > 2 \tag{4.26}$$

which for $\alpha = 3$ and 4 gives the values $\nu_{f1} = 0.68$ and 0.41. While these coefficients are rather large, they are associated with scatter values of fatigue life that are at the upper limit of those observed in tests of structural parts and therefore higher than those to be expected for full-scale structures. Moreover the assumption of an extreme-value distribution of fatigue lives for convenient analysis in closed form is quite unfavorable in view of the fact that existing observations can be reasonably well fitted by logarithmic-normal distribution. Thus it appears that in reality the coefficient of variation of f_1 might be smaller than suggested by the above values and that therefore a design with an expected value $0.2 < f_1 < 0.4$ may produce structures that can be classified as fatigue insensitive with a sufficiently high probability. Thus, for instance, it appears from Fig. 4 that even under the assumptions made a design with expected $E[f_1] =$

0.3 for $\alpha = 4$ and would exceed $f_1 = 1$ only with a probability of about 0.10 and $f_1 = 2$ with 0.01.

Obviously in actual design the problem arises whether a fatigue-insensitive design can, in fact, be attained with the material used; in other words whether the operational fatigue load spectrum applied to the structure produces dimensions which are compatible with those arising from the ultimate load design with a prescribed probability of failure P_F. For a given material and operating conditions such design may not be compatible with conventionally assumed ratios between ultimate and limit load; if, as a result a fatigue-sensitive design with $f_1 \gg 1$ has to be accepted the necessary provisions for fail-safe construction must be made which may not be unnecessary in a fatigue-insensitive design. The estimate of the safe life of such a structure with a specified reliability must be made on the basis of the distribution function $p(f_1)$. This subject will be dealt with in a future report.

REFERENCES

1. Freudenthal, Alfred, M., Technical Report AFML-TR-66-37, February 1966.

2. Eugène, J., Proc. 4th Congress, Int. Council Aeronautical Sciences, August 1964, Spartan Books- MacMillan 1965, p. 685 ff.

3. Freudenthal, A. M. and Shinozuka M., Structural Reliability, J. Wiley and Sons, New York, 1967.

4. Freudenthal, A. M. and Gumbel, E. J., Adv. Appl. Mech., Vol. 4 (1956) 132 ff.

5. Freudenthal, A. M. and Gumbel, E. J., J. Am. Statistical Assoc. Vol. 49 (1954) 575.

6. Freudenthal, A. M., Proc. Am. Soc. Test. Mat., Vol. 51 (1951) 583.

7. Freudenthal, A. M., Tech. Doc. Report No. ML-TDR-64-300, Sept. 1964.

8. Freudenthal, A. M., J. Struct. Div. Am. Soc. Civil Eng., Vol. 87 (1961) N ST3, 1.

9. Gumbel E. J., Statistics of Extremes, Columbia Univ. Press, New York (1958), 277.

10. Selichof, A., Proc. 4th Congress Int. Council Aeron. Sciences, August 1964, Spartan Books - MacMillan 1965, 815 ff.

11. Freudenthal, A. M. and Payne, A. O., Technical Report AFML-TR-64-401, December 1964.

12. Freudenthal, A. M., Garrelts, J. M. and Shinozuka, M., J. Struct. Div. Am. Soc. Civil Eng., Vol. 92, No. ST1, 280 ff.

13. Freudenthal, A. M. and Heller, R. A., J. Aerospace Sciences, Vol. 26 (1959) 431.

14. McCulloch, A. J., Am. Soc. Test. Mat. Special Techn. Publ. No. 274, p. 30 ff.

15. Schleicher, R., Fatigue in Aircraft Structures, Acad. Press, New York, 1957, 376 .

16. Freudenthal, A. M., WADD Technical Report 61-53, July 1961.

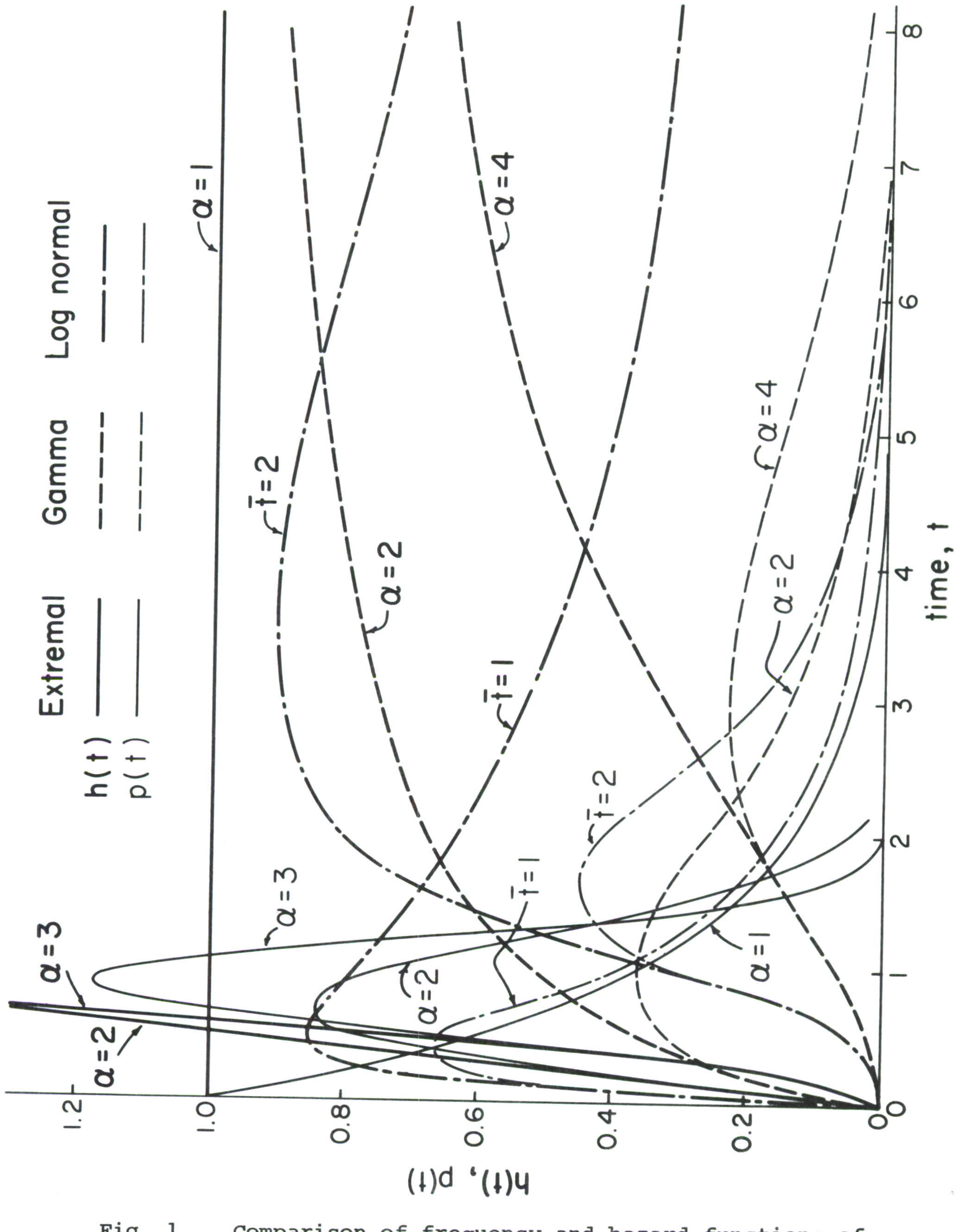

Fig. 1 Comparison of frequency and hazard functions of Gamma, Extremal and Log-Normal distributions.

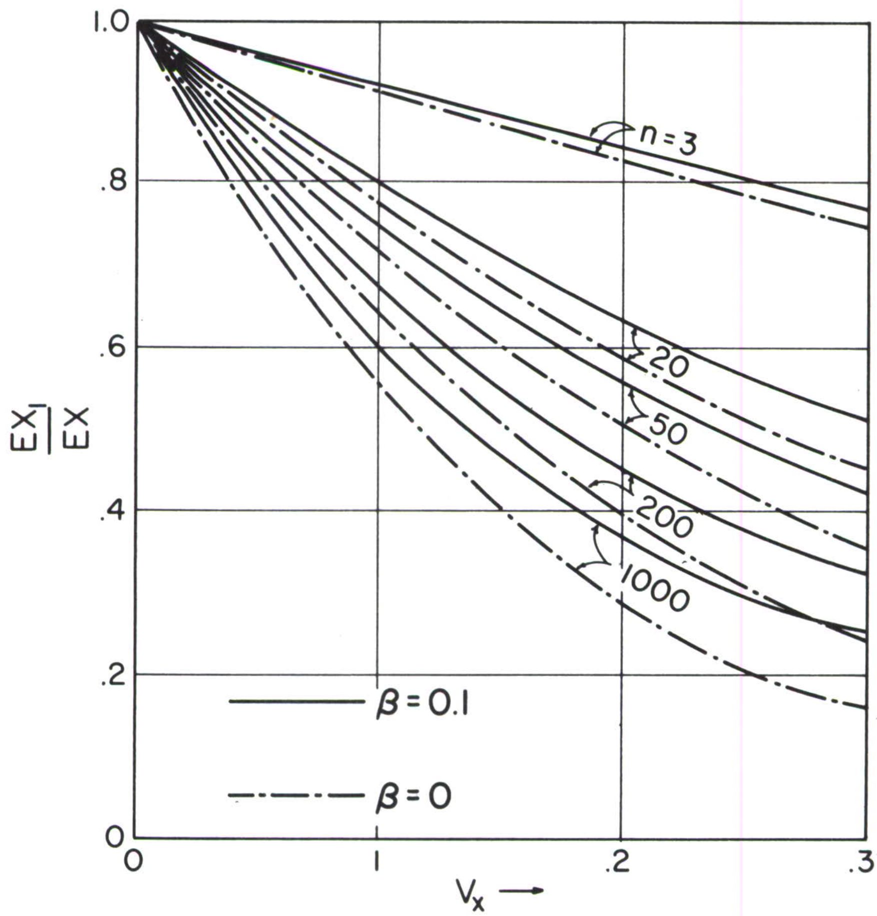

Fig. 2a Ratio EX_1/EX versus coefficient of variation V_x for $x_o = 0$ and $x_o = 0.1v$ and n = 3, 20, 50, 200 and 1000.

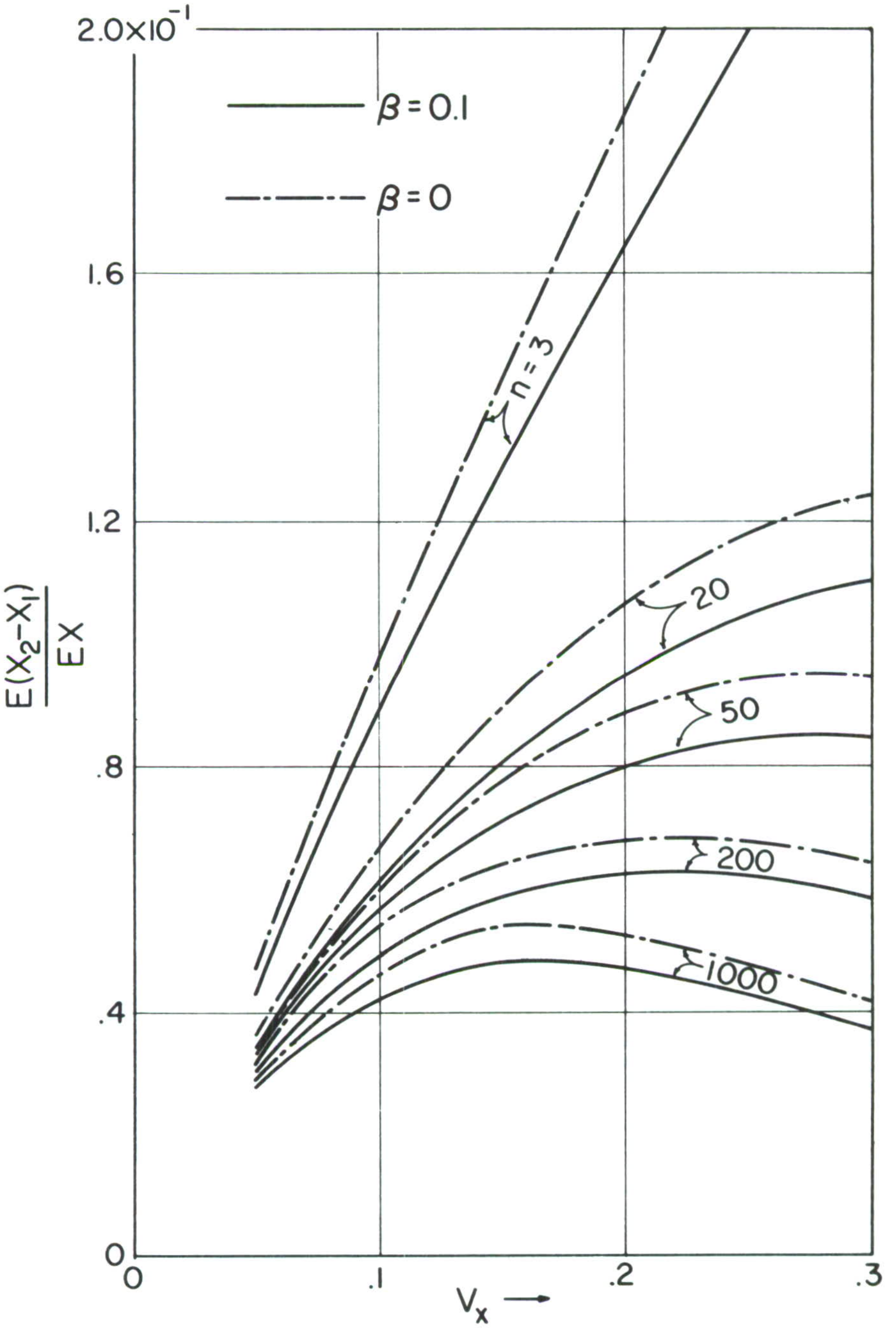

Fig. 2b Ratio $E(X_2-X_1)EX$ versus coefficient of variation V_x for $x_o = 0$ and $x_o = 0.1v$ and n = 3, 20, 50, 200 and 1000.

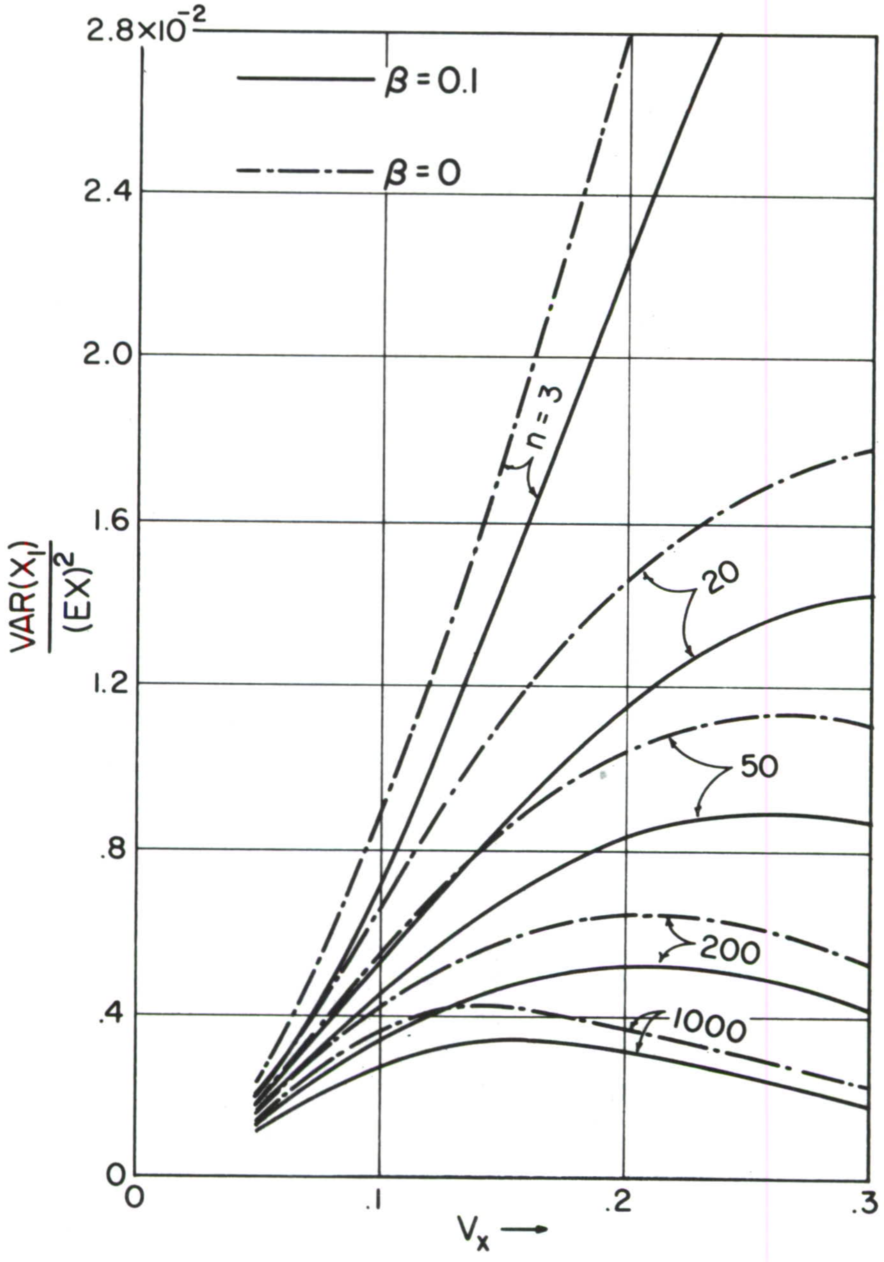

Fig. 2c Ratio Var $X_1/(EX)^2$ versus coefficient of variation V_x for $x_o = 0$ and $x_o = 0.1v$ for n = 3, 20, 50, 200 and 1000.

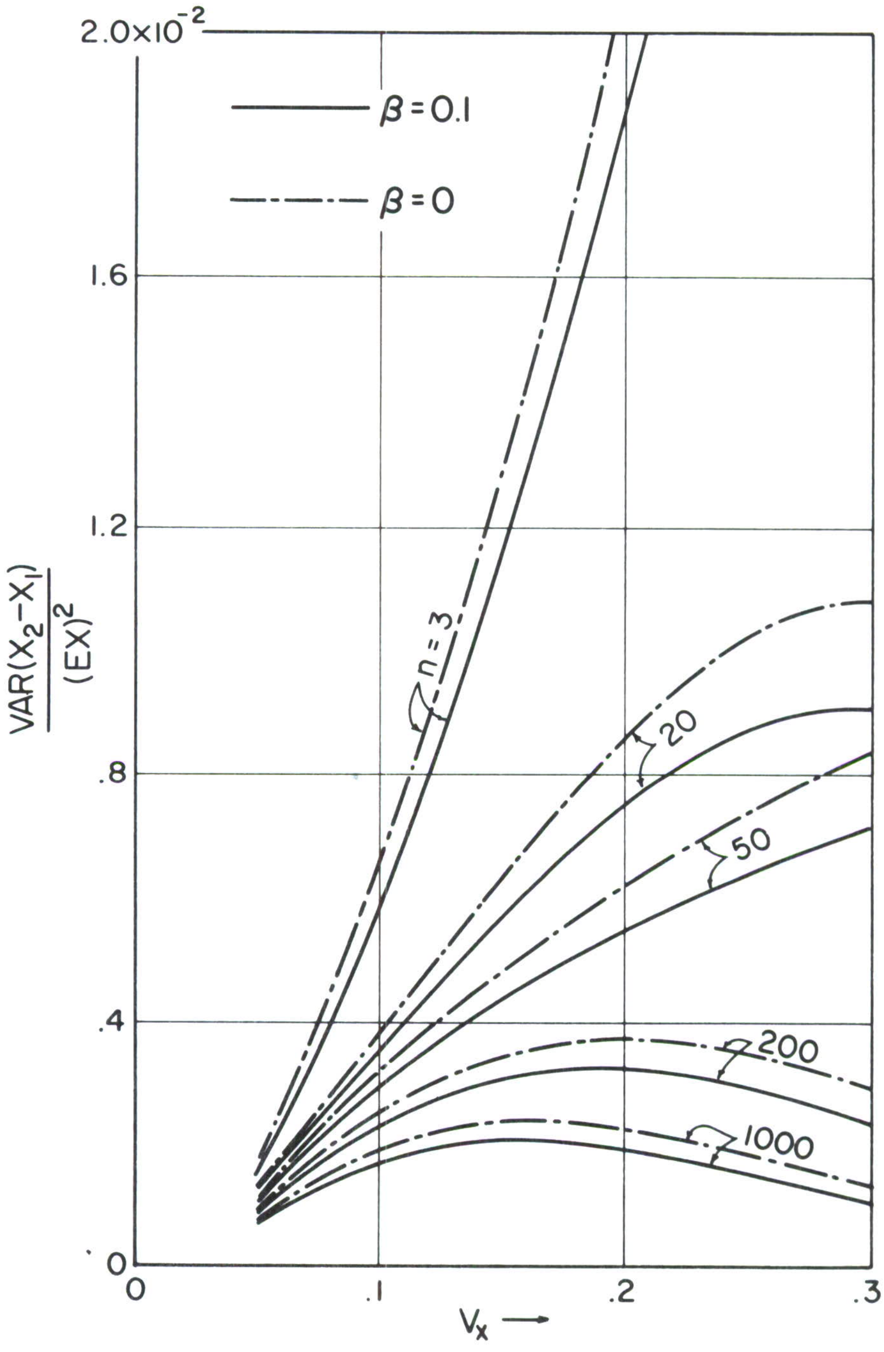

Fig. 2d Ratio $\mathrm{Var}(X_2-X_1)/(EX)^2$ versus coefficient of variation V_x for $x_o = 0$ and $x_o = 0.1v$ for n = 3, 20, 50, 200 and 1000.

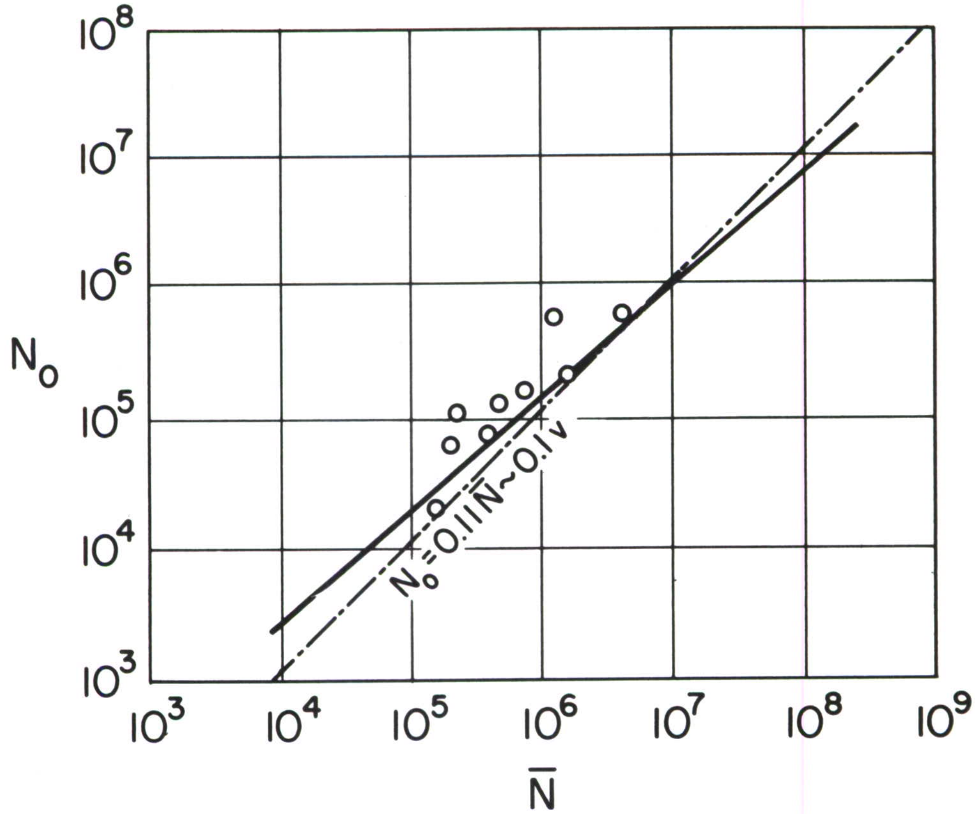

Fig. 3 Minimum life N_o' as function of mean life $\bar{N}$ (Ref. 10).

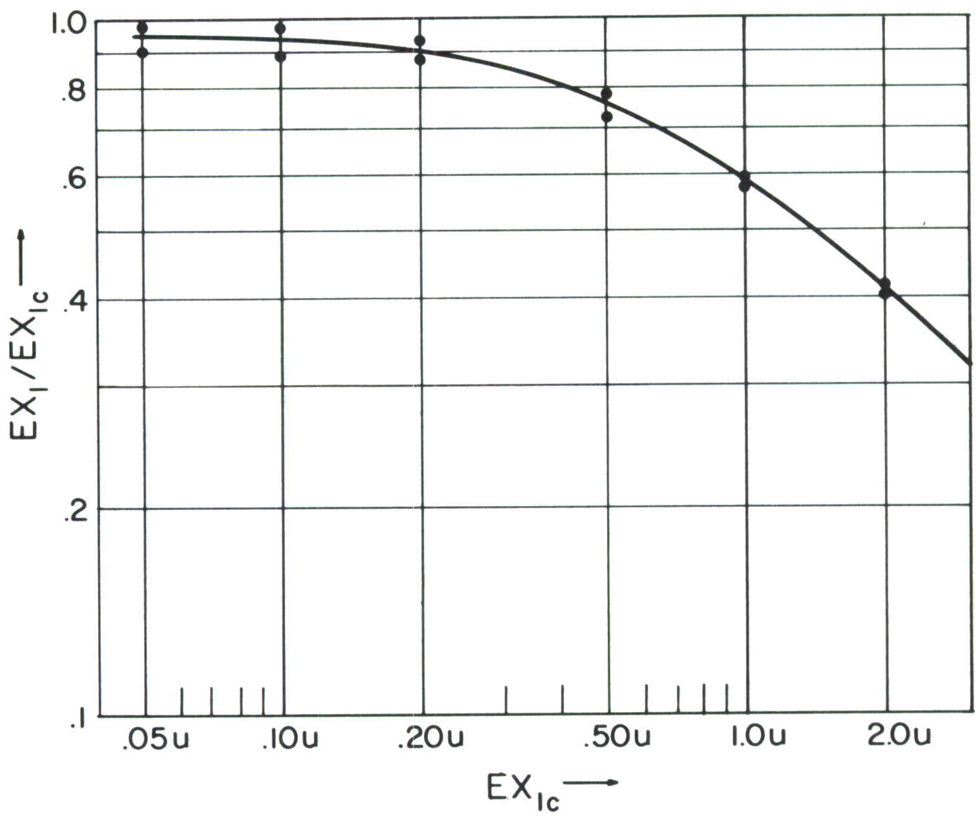

Fig. 4 Ratio EX_1/EX_{1C} versus EX_{1C} for reliability function Eq. (2.17).

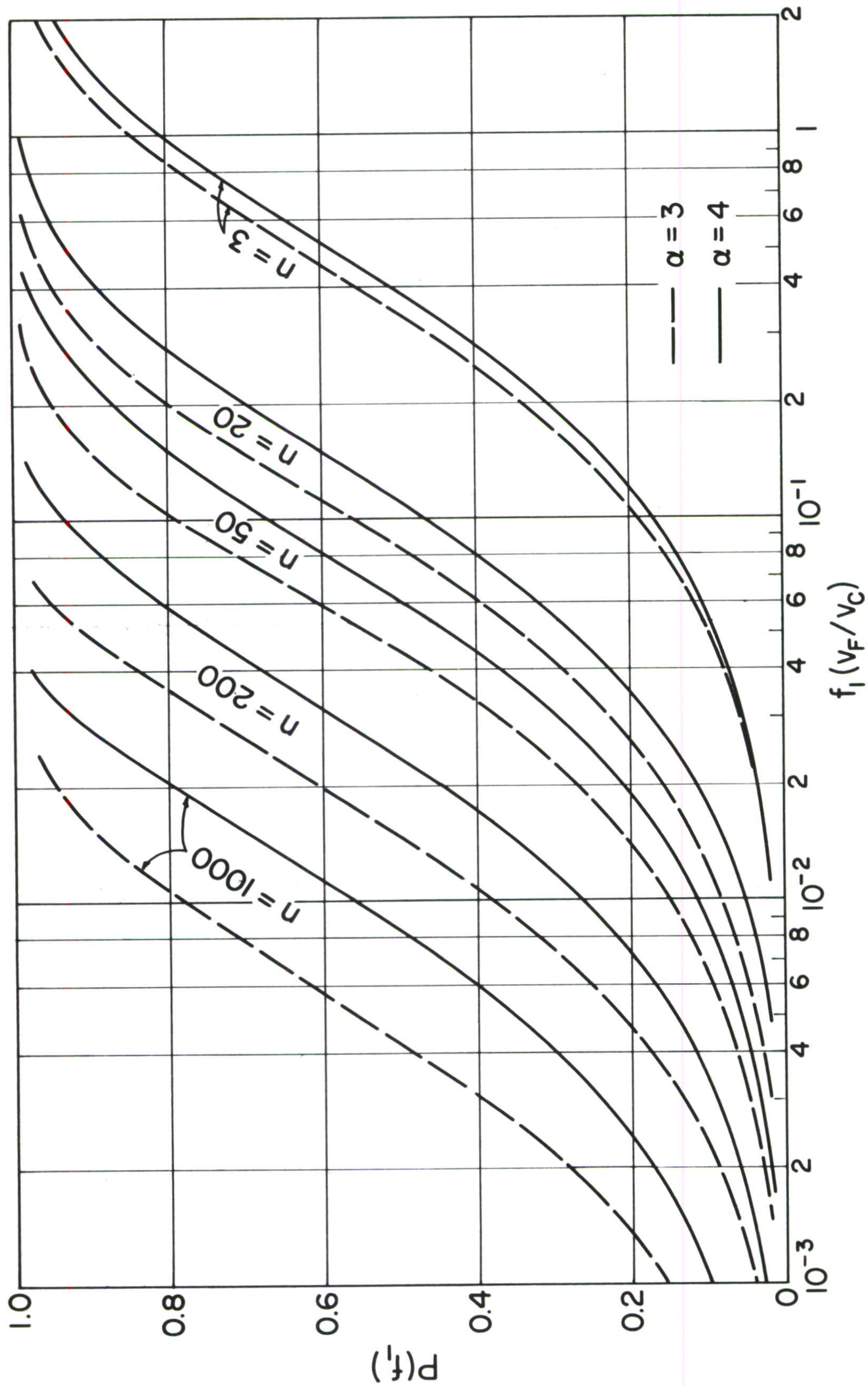

Fig. 5 Distribution function of the fatigue sensitivity factor $P(f_1)$ plotted versus $f_1(v_F/v_C)$ for $\alpha = 3$ and 4 and n = 3, 20, 50, 200 and 1000.

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Department of Civil Engineering and Engineering Mechanics, Columbia University New York, N. Y.	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

RELIABILITY ANALYSIS BASED ON TIME TO FIRST FAILURE

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Summary Report - April 15 to December 31, 1966.

5. AUTHOR(S) *(Last name, first name, initial)*

Freudenthal, A. M.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
1967	37	16

8a. CONTRACT OR GRANT NO. AF 33(675)8741	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO. 7351	AFML-TR-67-
c. Task No. 735106	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	

10. AVAILABILITY/LIMITATION NOTICES This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of the Metals and Ceramics Division (MAM), Air Force Materials Laboratory, Wright-Patterson AFB, Ohio.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Metals and Ceramics Division Air Force Materials Laboratory Wright-Patterson AFB, Ohio 45433

13. ABSTRACT

The new approach to structural reliability analysis based on order statistics and the expected time to first failure in a fleet of specified magnitude, that has been first proposed in a previous report AFML-TR-66-37 is developed in more detail and applied to structures subject to progressive fatigue damage. It is shown that the concept of fatigue sensitivity developed in WADD Tech. Rep. 61-53 is improved by relating it to the expected time to first failure rather than to the expected time to failure. The modified fatigue sensitivity factor becomes an effective parameter for the correlation of ultimate load and fatigue design and for the classification of fatigue sensitive structures.

This abstract is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of the Metals and Ceramics Division (MAM), Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio.

Unclassified

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2a. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7a. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8a. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through ________________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through ________________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through ________________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.